Optimisation Validation

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Abstract

We introduce the idea of *optimisation validation*, which is to formally establish that an instance of an optimising transformation indeed improves with respect to some resource measure. This is related to, but in contrast with, *translation validation*, which aims to establish that a particular instance of a transformation undertaken by an optimising compiler is semantics preserving. Our main setting is a program logic for a subset of Java bytecode, which is sound and complete for a resource-annotated operational semantics. The latter employs *resource algebras* for measuring dynamic costs such as time, space and more elaborate examples. We describe examples of optimisation validation that we have formally verified in Isabelle/HOL using the logic. We also introduce a type and effect system for measuring static costs such as code size, which is proved consistent with the operational semantics.

Key words: Compiler Optimisation, Translation Validation, Program Logic, Java Virtual Machine Language, Cost Modelling, Resource Algebras, Lightweight Verification.

1 Introduction

We are interested in certifying the resource usage of mobile code for the Java platform. In previous work [3,1,6] we have described a proof-carrying code

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infrastructure which does this for memory usage. A class file is accompanied by a proof certificate which describes the resource usage of the `main` method of the program; we use a program logic with judgments of the form $\rhd e : \{P\}$, stating that expression S satisfies assertion P. For example:

$$\rhd e_{\mathtt{main}} : \{r = F(h)\}$$

where h is the starting heap of the program (in particular, containing the arguments to the `main` method) and r is the memory consumption of the program expressed as a function of the size of the arguments in h.

In this paper we investigate a significant extension of this framework and a particular application. First, we generalise the form of resources so that a wider range of notions is covered, in a generic fashion. Second, we consider orderings on resources which allow us to talk about *optimisation validation*, in the sense that we can establish when one program consumes fewer resources than another. This is inspired by the idea of *translation validation* [22], an alternative to the wholescale verification of translators and compilers. In this approach, one instead constructs a validation mechanism that, after every run of a compiler, formally confirms that the target code produced on that run is a correct translation of the source producing:

> "[...] the same result while *(hopefully)* executing in less time or space or consuming less power." [23]

(our emphasis). Here, we take the reduction in resource usage as being the primary motivation, and therefore, what should be checked. This is appropriate in scenarios such as the safety policies considered in proof-carrying code, where resource usage may even be a more important concern than correctness, because it encompasses the security requirements of the domain.

Notions of optimisation. To consider validating optimisations, we must first define what optimisation is in our setting. We suppose that a program is given as a collection of classes, one of which includes a nominated `main` method. A simple notion of *dynamic* optimisation is with respect to every terminating execution of this method. Let P_1 be the program before optimisation and P_2 be the program after:

$$P_1 \longrightarrow P_2$$

We only need to consider the costs for the bodies of the main method in each program,

$$e_1 \longrightarrow e_2$$

(changes in other methods may be optimising, neutral or even non-optimising; at this point we do not study optimisations within nested program contexts). To be considered an optimisation, we want to establish that the transformation is improving with respect to a cost model. We capture the latter with the notion of *resource algebra* $\mathcal{R}$, which contains components for measuring the cost of executing each kind of instruction, along with an ordering on those costs. The overall (dynamic) cost may depend on the input of the program, and is measured by execution in a operational semantics annotated with calculations using $\mathcal{R}$. If for all input heaps both e_1 and e_2 converge, then the resource consumption of e_2 should improve on that of e_1 [4]:

$$h \vdash e_1 \Downarrow r_1 \quad \wedge \quad h \vdash e_2 \Downarrow r_2 \implies r_2 \leq r_1$$

where the ordering $\leq$ refers to the ordering from $\mathcal{R}$. We may assume, without loss of generality, that the input pointer for the argument to `main` is fixed on every execution.

Optimisation sequences. The above defines our notion of a single-step optimisation. For several optimisations in sequence, it is enough to consider an optimisation between the initial and final programs for the resource algebra of interest $\mathcal{R}$. However, we often want to decompose a sequence of optimisations into several transformations which are individually optimising. Then we can show the existence of a sequence of optimising steps:

$$P_1 \longrightarrow P_2 \longrightarrow \cdots \longrightarrow P_n$$

where each $P_i \longrightarrow P_{i+1}$ is an optimisation for some particular resource algebra $\mathcal{R}_i$. Additionally, each step in the optimisation should be non-increasing for the target cost model $\mathcal{R}$. A *proper* optimisation sequence has at least one step for which costs in $\mathcal{R}$ strictly decrease from some P_i to P_{i+1}.

Validating optimisations by program logic. To state and prove (dynamic) cost optimisations, we use a program logic to make assertions about functions that bound the resource consumed. We must find assertions of the form:

$$ST_1 \rhd e_1 \;:\; \{F_1(h) \leq r\} \qquad\qquad ST_2 \rhd e_2 \;:\; \{r \leq F_2(h)\}$$

where the *specification tables* ST_i associate an assertion to each method and function in the program, providing the appropriate invariant. The assertions state that the resource consumed when executing P_1 is bounded from below by some function F_1 of the input heap, and that the resources consumed by P_2 are bounded from above by a function F_2. To show that P_2 is an optimisation of P_1 we must now prove that:

$$\forall h.\; F_2(h) \leq F_1(h)$$

(in particular, this holds trivially in case $F_1 = F_2$).

[4] Cf. Sands's *Improvement Theory* [24] and his intensional notion of cost equivalence.

Static optimisations. Static costs such as code size are commonly used as metrics for optimisation and some dynamic costs can be usefully approximated with static measurements. We cover both possibilities by introducing a notion of *static* resource algebra $\mathcal{S}$. To measure static costs, we use a type system with effects. For two function bodies e_1 and e_2 we must find a type t and effects s_1 and s_2 such that:

$$\Gamma_{\mathtt{main}} \vdash_{\Sigma_1} e_1 \ : \ t, s_1 \qquad\qquad \Gamma_{\mathtt{main}} \vdash_{\Sigma_2} e_2 \ : \ t, s_2$$

where the static typing context for the body of main has the form $\Gamma_{\mathtt{main}} = \mathtt{args} : \mathtt{String}[]$ and Σ_1 and Σ_2 are the resource typing signatures of programs P_1 and P_2 respectively, see Sect. 5.

For P_2 to be a static optimisation of P_1 we should establish that $s_2 \leq s_1$, where the ordering $\leq$ now refers to the ordering on static costs. An ideal notion of optimisation would be w.r.t. a pair $(\mathcal{R}, \mathcal{S})$ of target dynamic and static cost models; a sequence of optimisations might alternate dynamic and static reductions as appropriate. A typical example is to use time *and* code size to validate optimisations such as loop unrolling, see Sect.4.1. To simplify exposition here, we consider the costs separately.

This paper is organised as follows. In Sect. 2 we present the dynamic semantics of our language, introduce resource algebras, and describe some typical instantiations. In Sect. 3, we present a program logic that generalises the logic presented in [1] to arbitrary resource algebras. Sect. 4 gives example optimisation validations, including standard compiler optimisation steps and tail-call optimisation. An application specific example is considered in App. A.1, together with full listing of operational semantics, program logic and typing rules. Sect. 5 examines the static system. Finally, Sect. 6 concludes with a summary and discussion of related work.

2 Resource annotated operational semantics

We use a functional form of Java bytecode called Grail [7], although the approach would work for other languages endowed with a structural operational semantics. Grail retains the object and method structure of JVML, but represents method bodies as sets of mutually tail-recursive first-order functions. The language is built up from values v, arguments a, and function body expressions e (in this paper we do not mention static fields and virtual invocation, which are accounted for elsewhere [1]):

$$\begin{aligned}
v &::= () \mid l_C \mid i \mid \mathsf{null}_C \\
a &::= v \mid x \\
e &::= a \mid \mathsf{prim}\ a\ a \mid \mathsf{new}\ C \mid x.f \mid x.f := a \mid e\ ;\ e \mid \mathsf{let}\ x = e\ \mathsf{in}\ e \\
&\quad \mid \mathsf{if}\ e\ \mathsf{then}\ e\ \mathsf{else}\ e \mid \mathsf{call}\ g \mid C.m(\overline{a})
\end{aligned}$$

Here, C ranges over Java class names, f over field names, m over method names, x over variables (method parameters and locals) and g over function names (which correspond to instruction addresses in bytecode). Values consist of integer constants i, typed locations l_C, the unique element () of type unit and the nullary reference null_C. As in JVML, the booleans bool are defined as $\mathsf{true} \overset{def}{=} 1$, $\mathsf{false} \overset{def}{=} 0$.

The (impure) call-by-value functional semantics of Grail coincides with an imperative interpretation of its direct translation into JVML, provided some syntactic conditions are met. In particular, actual arguments in function calls must coincide with the formal parameters of the function definitions. Example Grail programs are shown in Fig. 1 and 2.

To model consumption of computational resources, our semantics is annotated with a resource counting mechanism based on *resource algebras.*

Definition 2.1 A *resource algebra* $\mathcal{R}$ is a partially ordered monoid $(R, 0, +, \leq)$, i.e. $(R, 0, +)$ is a monoid and $(R, \leq)$ a partially ordered set, where 0 is the minimum element, and + is order preserving on both sides. Moreover, $\mathcal{R}$ has constants in R for each expression former: $\mathcal{R}^{\mathsf{int}}$, $\mathcal{R}^{\mathsf{null}}$, $\mathcal{R}^{\mathsf{var}}$, $\mathcal{R}^{\mathsf{prim}}$, $\mathcal{R}^{\mathsf{new}}_C$, $\mathcal{R}^{\mathsf{getf}}$, $\mathcal{R}^{\mathsf{putf}}$, $\mathcal{R}^{\mathsf{comp}}$, $\mathcal{R}^{\mathsf{let}}$, $\mathcal{R}^{\mathsf{if}}$, $\mathcal{R}^{\mathsf{call}}$ and a monotone operator $\mathcal{R}^{\mathsf{meth}}_{C,m,\overline{v}} : R \to R$.

Each constant denotes the cost associated to an instruction, which are then composed via the monoidal operation. The operator $\mathcal{R}^{\mathsf{meth}}$ calculates a cost for method calls. For some applications, we might parameterise the constants with additional pieces of syntax, for example if we are tracking read/writes of certain variables or charge selected primitive operations differently.

The operational semantics defines a judgement

$$E \vdash h, e \Downarrow h', v, r$$

which relates expressions e to environments E (maps from variables to values), initial and final heaps h, h', result values v and costs $r \in R$. Heaps are partial maps from locations l to objects, where an object is represented as a class name C together with a field table (a map from field names f to values v). An example rule is that for evaluating the true branch of a conditional:

$$\frac{E \vdash h, e \Downarrow h', \mathtt{1}, r_e \qquad E \vdash h', e_1 \Downarrow h'', v, r}{E \vdash h, \mathsf{if}\ e\ \mathsf{then}\ e_1\ \mathsf{else}\ e_2 \Downarrow h'', v, r_e + \mathcal{R}^{\mathsf{if}} + r}$$

This is the familiar rule augmented with the resource calculation using the $\mathcal{R}^{\mathsf{if}}$ constant. For all the resource algebras considered here, '+' is commutative; however, for examples where it is not, the order of '+' in the rules is important and matches the evaluation order. The full definition of the semantics appears in Appendix A.2.

	Time	Heap	Frames	MethCnts	$\mathsf{MethFreq}^{Id}$	MethGuard
$\lvert\mathcal{R}\rvert$	$\mathcal{N}$	$\mathcal{N}$	$\mathcal{N}$	$\mathcal{MS}(Id)$	$\mathcal{N}\times\mathcal{N}$	$\{\mathsf{tt},\mathsf{ff}\}$
$\mathcal{R}^{\mathsf{int}}$	1	0	0	$\emptyset$	$(1,0)$	tt
$\mathcal{R}^{\mathsf{null}}$	1	0	0	$\emptyset$	$(1,0)$	tt
$\mathcal{R}^{\mathsf{var}}$	1	0	0	$\emptyset$	$(1,0)$	tt
$\mathcal{R}^{\mathsf{prim}}$	1	0	0	$\emptyset$	$(1,0)$	tt
$\mathcal{R}^{\mathsf{new}}_{C}$	3	$size(C)$	0	$\emptyset$	$(3,0)$	tt
$\mathcal{R}^{\mathsf{getf}}$	2	0	0	$\emptyset$	$(2,0)$	tt
$\mathcal{R}^{\mathsf{putf}}$	3	0	0	$\emptyset$	$(3,0)$	tt
$\mathcal{R}^{\mathsf{comp}}$	0	0	0	$\emptyset$	$(0,0)$	tt
$\mathcal{R}^{\mathsf{let}}$	1	0	0	$\emptyset$	$(1,0)$	tt
$\mathcal{R}^{\mathsf{if}}$	0	0	0	$\emptyset$	$(0,0)$	tt
$\mathcal{R}^{\mathsf{call}}$	1	0	0	$\emptyset$	$(1,0)$	tt
$\mathcal{R}^{\mathsf{meth}}_{C,m,\overline{v}}(r)$	$\lvert\overline{v}\rvert+2+r$	r	$r+1$	$r\cup_{+}\{C.m\}$	$\mathsf{Freq}_{C.m,\lvert\overline{v}\rvert}(r)$	$G_{C,m}(\overline{v})\wedge r$
$0_{\mathcal{R}}$	0	0	0	$\emptyset$	$(0,0)$	tt
$+_{\mathcal{R}}$	$+$	$+$	max	$\cup_{+}$	$+_{\mathsf{Freq}}$	$\wedge$
$\leq_{\mathcal{R}}$	$\leq$	$\leq$	$\leq$	$\subseteq_{+}$	$\leq_{\mathsf{Freq}}$	$\leq_{\mathsf{Guard}}$

The notation $\lvert\overline{v}\rvert$ denotes the length of the list $v_1 \ldots v_n$. For method counts, $\cup_{+}$ and $\subseteq_{+}$ are multiset union and subset respectively. For frequencies, we define $\mathsf{Freq}_{Id,n}(t,p) = (0, max(t,p))$ and $\mathsf{Freq}_{C.m,n}(t,p) = (n+2+t, p)$ for $C.m \neq Id$. Composition in this case is $(t,p)+_{\mathsf{Freq}}(t',p') = (t+t', max(p,p'))$ and the ordering $(t,p) \leq_{\mathsf{Freq}} (t',p')$ iff $p \leq p'$. For guards, $G_{C,m}(\overline{v})$ is a boolean valued function for each C, m and $b \leq_{\mathsf{Guard}} b'$ iff $b = \mathsf{tt}$ or $b = b' = \mathsf{ff}$.

Table 1
Example resource algebras

2.1 *Resource algebra examples*

Some example resource algebras are shown in Table 1. The Time algebra models an instruction counter that approximates execution time; each Grail expression form is charged according to the number of JVM instructions to which it expands[5]. The Heap algebra counts the size of heap space consumed during execution (ignoring the possibility of garbage collection, which cannot be assumed for an arbitrary JVM). Only the *new* instruction consumes heap. The Frames algebra counts the maximal number of frames on the stack during execution. The MethCnts algebra traces invocations by accumulating a multiset of invoked method names.

The $\mathsf{MethFreq}^{Id}$ algebra calculates a measure of the frequency of calls to the method *Id* (a long identifier *C.m*), by accumulating the maximal period

[5] There are no costs for the *if* instructions because they are compiled as test and branches; similarly, sequential composition has no cost in these example algebras.

between successive calls; this is an example of an application specific algebra (see App. A.1 for a motivating example).

Finally, the $\mathsf{MethGuard}$ algebra does not calculate a quantitative resource, but rather maintains a boolean monitor that checks that arbitrary guards $G_{C,m}(\overline{v})$ are satisfied at invocations of method m in class C. If guards are considered as resource usability preconditions (for example, to check that a method parameter lies within some limits), then we may consider an optimisation to be a transformation which ensures the resource preconditions are always satisfied.

In this last case, the resource operator $\mathcal{R}^{\mathsf{meth}}_{C,m,\overline{v}}$ depends on the run-time values v_i, whereas in the other examples the function is fixed because the length of the argument list is specified by the definition of the method $C.m$. In general, resource algebras that depend on runtime values such as this can collect traces along the path of computation; the resulting word may be constrained by further policies, specified for example by security automata [25] or by formulae from logics over linear structures, which can be encoded in the higher-order assertion language of our program logic, introduced next.

3 Resource-aware program logic

Our primary basis for optimisation validation is a general-purpose program logic for Grail where assertions are boolean functions over all semantic components occurring in the operational semantics, namely the input environment E and initial heap h, and the post heap h', the result value v, and the resources consumed r. An assertion P thus belongs to the type $\mathcal{E} \to \mathcal{H} \to \mathcal{H} \to \mathcal{V} \to \mathcal{R} \to \mathcal{B}$. A judgement $G \rhd e : P$ in the logic relates a Grail expression e to an assertion P, dependent on a context $G = \{(e_1, P_1), \ldots, \{e_n, P_n)\}$ that stores assumptions for recursive program structures, in the spirit of Hoare's original proof rule for procedures [16]. The program logic comprises one rule for each expression form, an axiom and a consequence rule, see Appendix A.3 for the complete listing and Appendix A.2 for heap-related notation. Here we show the getfield rule, where free variables in assertions in braces are universally quantified.

$$\frac{}{G \rhd x.f : \{h = h' \wedge (\exists l. E(x) = l \wedge v = h(l).f) \wedge r = \mathit{cost}(x) + \mathcal{R}^{\mathsf{getf}}\}}$$

Before demonstrating how the program logic is used to verify the resource consumption of programs, we summarise some basic meta-theoretical properties. These have been formally proven by representing the operational semantics and the program logic in the proof assistant Isabelle/HOL; for more details, see [1,2]; the results here are a mild generalisation, namely with resource algebras, of those presented there. First,semantic validity, which has a partial correctness interpretation:

Definition 3.1 An assertion P is *valid* for expression e, written $\models e : P$, if for all E, h, h', v and r $E \vdash h, e \Downarrow h', v, r$ implies the assertion $(P\ E\ h\ h'\ v\ r)$

holds. A context G is *valid*, $\models G$, if for all pairs (e, P) in G, it holds that $\models e : P$. Assertion P is *valid for e in context G*, if $\models G$ implies $\models e : P$.

Indeed, the proof system is sound with respect to the operational semantics:

Theorem 3.2 (Soundness) *If $G \rhd e : P$ then $G \models e : P$.*

Because of the partial correctness interpretation, non-terminating programs satisfy their specifications vacuously. To verify resource consumption of possibly non-terminating programs, we can use an auxiliary termination logic, see [2].

The treatment of logical completeness as well as the concrete proving methodology benefits from some admissible rules concerning the proof context G. Beyond the usual weakening rule, other rules allow one to discharge the proof context, i.e. to derive judgements in the absence of contextual assumptions. This is done using a *specification table* ST, which maps function and method calls into assertions.

$$\frac{ST \models G \quad (e, P) \in G}{\emptyset \rhd e : P} \qquad \frac{ST \models G \quad (C.m(\overline{a}), ST(C, m, \overline{a})) \in G}{\emptyset \rhd C.m(\overline{b}) : ST(C, m, \overline{b})}$$

The rule shown on the left (SPECTABLE) allows one to verify (possibly mutually recursive) program fragments using the specification table, while the right rule allows one to *adapt* the actual arguments when extracting method specifications. A context G *respects* the specification table ST, notation $ST \models G$, if all entries consist of a function or method call together with its assertion in the table; moreover their bodies satisfy a corresponding assertion.

To prove relative completeness, we define a context G_{strong} that associates to each function and method call its strongest specification.

Definition 3.3 The strongest specification for e is $SSpec(e) \equiv \{E \vdash h, e \Downarrow h', v, r\}$.

Lemma 3.4 *For any e, $G_{strong} \rhd e : SSpec(e)$.*

Furthermore, G_{strong} satisfies $ST_{strong} \models G_{strong}$, where ST_{strong} is the specification table defined by $ST_{strong} \equiv (\lambda g.\ SSpec(\mathsf{call}\ g), \lambda C m \overline{a}.\ SSpec(C.m(\overline{a})))$. From this, we obtain:

Theorem 3.5 (Completeness) *For any e and $P \models e : P$ implies $\emptyset \rhd e : P$.*

The completeness result means that we can conceivably prove any provable assertion using the rules of the program logic, following the structure of the program.

4 Validated optimisations

The program logic presented in the previous section can be used to justify program transformations that are routinely applied in optimising compilers, *provided* they are in fact improving. In this section we give some example optimisations and sketch the proofs of their validation. While the transformations and the examples we consider in this paper are fairly simple, they serve the purpose of demonstrating our methodology.

4.1 Standard low-level optimisations

We first consider the motivating program of [23],

```
i <- 0; x <- 1; y <- 2;
WHILE i < 24 DO {i <- i + x + y ; g <- 2 * i}
EXIT
```

Our formal verification refers to a translation of this code into (the Isabelle representation of) our language. The result of this (manual) translation is the method $\texttt{R.calc}_0$

$$\begin{array}{l}
\texttt{method static int R.calc}_0\texttt{()} \;=\\
\quad \mathsf{let}\ i = 0\ \mathsf{in}\ \mathsf{let}\ x = 1\ \mathsf{in}\ \mathsf{let}\ y = 2\ \mathsf{in}\ \mathsf{let}\ g = 0\ \mathsf{in}\ \mathsf{call}\ f\\
\quad \texttt{fun f(int i, int x, int y, int g)} \;=\; \mathsf{if}\ i < 24\ \mathsf{then}\ \mathsf{call}\ h\ \mathsf{else}\ \mathsf{var}\ g\\
\quad \texttt{fun h(int i, int x, int y, int g)} \;=\\
\qquad \mathsf{let}\ j = i + x\ \mathsf{in}\ \mathsf{let}\ i = j + y\ \mathsf{in}\ \mathsf{let}\ g = 2 * i\ \mathsf{in}\ \mathsf{call}\ f
\end{array}$$

which differs from the original code only in minor ways: we extended the loop prelude by an assignment to variable g, converted the loop into two functions which represent basic blocks, and turned the EXIT statement into a return statement of the final value of g.

Using the resource algebra Time defined previously, we now outline our Isabelle proof of the judgement

$$\emptyset \rhd \texttt{R.calc}_0([\,]) : \{r = 213\} \tag{1}$$

which states that an invocation of $\texttt{R.calc}_0$ requires 213 units of time. We first define two auxiliary (semantic) functions

$$\begin{array}{l}
\mathit{cost}_f(n) = 24 * n + 10\\
\mathit{cost}_h(n) = 24 * n + 1
\end{array}$$

that describe the costs of evaluating functions f and h, respectively, where n is the number of loop iterations. Next, we define a specification table ST_0 that contains the specification of $\texttt{calc}_0$, and its local functions f and h.

$$ST_0 = \left[\begin{array}{l}
\texttt{R.calc}_0 \mapsto \{r = 11 + \mathit{cost}_f(8)\}\\
\quad \mathsf{call}\ f \mapsto \{\forall\, J.\ (E(x) = 1 \wedge E(y) = 2 \wedge E(i) = 3 * J \wedge J \leq 8) \longrightarrow\\
\qquad\qquad\qquad r = \mathit{cost}_f(8 - J)\}\\
\quad \mathsf{call}\ h \mapsto \{\forall\, J.\ (E(x) = 1 \wedge E(y) = 2 \wedge E(i) = 3 * J \wedge J \leq 7) \longrightarrow\\
\qquad\qquad\qquad r = \mathit{cost}_h(8 - J)\}
\end{array}\right]$$

The first line defines the specification of $\texttt{R.calc}_0$ directly in terms of the auxiliary function cost_f, while the entries for call f and call h ensure that the auxiliary functions correctly model the costs of the executing the local functions. In both cases, the specifications depend on the value of variable i; intuitively, the universally quantified variable J represents the number of loop iterations that have already been performed.

Next, we define a context, G_0 that associates the specification table entries to the relevant function and method calls, e.g.

$$G_0 = \{(\texttt{R.calc}_0([\,]),\ ST_0\ \texttt{R.calc}_0), (\mathsf{call}\ f, ST_0\ \mathsf{call}\ f), (\mathsf{call}\ h, ST_0\ \mathsf{call}\ h)\}$$

The core of the verification consists of establishing $ST_0 \models G_0$. This requires us to prove that each entry in G_0 is justified: for each entry $(\mathsf{call}\ f, A)$ – and similarly for method entries – we need to show that the body body_f satisfies $G_0 \rhd \mathit{body}_f : A[E, h, h', v, \mathcal{R}_f^{\mathsf{call}} + r]$, where the notation $A[E, h, h', v, r]$ indicates the instantiation of a predicate. Using the rules of our program logic, these proofs proceed syntax-directed in a verification-condition-generating fashion, leaving side conditions involving numeric constraints. The verification conditions are currently discharged by manual interaction with the theorem prover that involves case-splits and quantifier instantiations. Current research is investigating how discharge of those verification conditions could be delegated to fully automated (external) solvers.

Finally, having established $ST_0 \models G_0$, our Isabelle proof of (1) concludes with an application of the SPECTABLE rule.

In the same fashion, we have produced Isabelle proofs of specifications $\emptyset \rhd \texttt{R.calc}_i([\,]) : \{r = r_i\}$ for methods $\texttt{R.calc}_1 \ldots \texttt{R.calc}_7$ which arise from applying the code transformations described in [23] to $\texttt{R.calc}_0$. The resulting code is shown in Fig. 1, while the following table summarises the costs r_i obtained for each transformation step.

i	t_i	Transformation
0	213	
1	197	Constant propagation and constant folding
2	193	Dead assignment elimination
3	176	Branch movement, inlining, redundant test elimination
4	126	Induction variable elimination
5	126	Loop unrolling *without* code sharing
6	82	Dead code elimination
7	66	Expression folding

In general, proofs of functional correctness of arbitrary code fragments may be required to verify statements about resource consumption. However, this is not the case for the specific transformations we considered: none of the specifications involved constrains the result values v. Furthermore, none of the transformations increases the dynamic resources consumed. Indeed, except for loop unrolling (the conversion $\texttt{calc}_4 \to \texttt{calc}_5$), all transformations

```
class R {
...
method static int calc_1() = let $i = 0$ in let $x = 1$ in let $y = 2$ in let $g = 0$ in call $f$
  fun f(int i, int x, int y, int g) = if $i < 24$ then call $h$ else var $g$
  fun h(int i, int x, int y, int g) = let $i = i + 3$ in let $g = 2 * i$ in call $f$

method static int calc_2() = let $i = 0$ in let $g = 0$ in call $f$
  fun f(int i, int g) = if $i < 24$ then call $h$ else var $g$
  fun h(int i, int g) = let $i = i + 3$ in let $g = 2 * i$ in call $f$

method static int calc_3() = let $i = 0$ in let $g = 0$ in call $h$
  fun h(int i, int g) = let $i = i + 3$ in let $g = 2 * i$ in if $i < 24$ then call $h$ else var $g$

method static int calc_4() = let $g = 0$ in call $h$
  fun h(int g) = let $g = g + 6$ in if $g < 48$ then call $h$ else var $g$

method static int calc_5() = let $g = 0$ in call $h$
  fun f(int g) = let $g = g + 6$ in if $g < 48$ then call $h$ else var $g$
  fun h(int g) = let $g = g + 6$ in if $g < 48$ then call $f$ else var $g$

method static int calc_6() = let $g = 0$ in call $h$
  fun h(int g) = let $g = g + 6$ in let $g = g + 6$ in if $g < 48$ then call $h$ else var $g$

method static int calc_7() = let $g = 0$ in call $h$
  fun h(int g) = let $g = g + 12$ in if $g < 48$ then call $h$ else var $g$}
```

Fig. 1. A sequence of low level transformations

reduce the costs[6].

4.2 Tail-call optimisation

Next, we consider a recursive program involving heap structures. Figure 2 defines the class REV with method App for appending an element to a list, and methods $\mathtt{Rev}_1, \ldots, \mathtt{Rev}_3$ for reversing a list. We assume that objects of class LIST contain fields HD and TL of type int and LIST, respectively.

Concentrating our attention on the required height of the frame stack, we observe that method $\mathtt{Rev}_1$ is formulated using method recursion and employs the auxiliary method App. As all its recursive invocations are nested, $\mathtt{Rev}_1$

[6] The loop unrolling performed in [23] actually *increases* the dynamic costs, because it jumps to a shared code block instead of duplicating the continuation code. Using our formalism of static resources we could characterise this as a static optimisation instead, namely reducing code size.

```
class REV {
method static LIST App(LIST l, int i) = call app
   fun app(LIST l, int i) = if l = null then call app_0 else call app_1
   fun app_0(int i) = let l = null in let x = new LIST in
                      x.HD:=i ; x.TL:=l ; var x
   fun app_1(LIST l, int i) = let h = l.HD in let t = l.TL in
                              let t = REV.App(t, i) in l.TL:=t ; var l

method static LIST Rev_1(LIST l) = call rev1
   fun rev1(LIST l) = if l = null then null else call rev1_1
   fun rev1_1(LIST l) = let h = l.HD in let t = l.TL in
                        let t = REV.Rev_1(t) in REV.App(t, h)

method static LIST Rev_2(LIST l, LIST acc) = call rev2
   fun rev2(LIST l, LIST acc) = if l = null then var acc else call rev2_1
   fun rev2_1(LIST l, LIST acc) = let h = l.HD in let t = l.TL in
                                  l.TL:=acc ; REV.Rev_2(t, l)

method static LIST Rev_3(LIST l, LIST acc) = call rev3
   fun rev3(LIST l, LIST acc) = if l = null then var acc else call rev3_1
   fun rev3_1(LIST l, LIST acc) = let t = l.TL in l.TL:=acc ;
                                  let acc = var l in let l = var t in call rev3}
```

Fig. 2. Class REV

requires a frame stack of a height that depends linearly on the length of the input list. To express this dependency we define a predicate $h, v \models_X n$ that specifies when a reference value v represents a non-cyclic (integer) list of length n in a heap region $h\downharpoonright_X$.

$$\begin{aligned} h, v \models_{\emptyset} 0 \ &\equiv\ v = \mathsf{null} \\ h, v \models_{v \uplus Y} (n+1) \ &\equiv\ v \in \mathit{dom}(h) \wedge h(v) = \mathsf{LIST} \wedge h(v).\mathsf{TL} = t \wedge h, t \models_Y n \end{aligned}$$

We can now prove a specification that relates the length of the list to the stack depth:

$$\rhd\ \mathsf{REV}.\mathtt{Rev1}([a]) : \{\forall\ n\ X.\ h, E(a) \models_X n \longrightarrow r_{\mathsf{Frames}} = n+1\}$$

Method $\mathtt{Rev}_2$ arises from $\mathtt{Rev}_1$ by introducing an accumulator which allows us to eliminate the invocation of App and formulate the recursion as tail recursion. Its specification imposes well-structuredness conditions on both arguments: pointers must represent lists, which moreover should be *non-overlapping* in the heap. The frame depth depends only on the length of the first argument, which gives the same overall depth cost as $\mathtt{Rev}_1$ (but considerable saving in

$$\left[\begin{array}{rl}
\mathsf{REV.}\mathtt{App}(\overline{a}) \mapsto & \{\forall\ x\ y\ n\ X. \\
& \quad (\overline{a} = [x,y] \wedge h, E(x) \models_X n) \longrightarrow \\
& \quad\quad (h', v \models_{\{\mathsf{freshloc}(h)\} \cup X} n+1 \wedge h =_{dom(h)\setminus X} h' \wedge r = r_{\mathtt{App}}(n))\} \\
\mathsf{REV.}\mathtt{Rev}_1(\overline{a}) \mapsto & \{\forall\ x\ n\ X. \\
& \quad (\overline{a} = [x] \wedge h, E(x) \models_X n) \longrightarrow (\exists\ Y.\ h', v \models_Y n \wedge r = r_{\mathtt{Rev1}}(n))\} \\
\mathsf{REV.}\mathtt{Rev}_2(\overline{a}) \mapsto & \{\forall\ x\ n\ X\ y\ m\ Y. \\
& \quad (\overline{a} = [x,y] \wedge h, E(x) \models_X n \wedge h, E(y) \models_Y m \wedge X \cap Y = \emptyset) \longrightarrow \\
& \quad\quad (\exists Z.\ h', v \models_Z n+m \wedge r = r_{\mathtt{Rev2}}(n))\} \\
\mathsf{REV.}\mathtt{Rev}_3(\overline{a}) \mapsto & \{\forall\ x\ n\ X\ y\ m\ Y. \\
& \quad (\overline{a} = [x,y] \wedge h, E(x) \models_X n \wedge h, E(y) \models_Y m \wedge X \cap Y = \emptyset) \longrightarrow \\
& \quad\quad (\exists Z. h', r \models_Z n+m \wedge r = r_{\mathtt{Rev4}}(n))\} \\
\mathsf{call}\ \mathtt{rev3} \mapsto & \{\forall\ n\ X\ m\ Y. \\
& \quad (h, E(\mathtt{l}) \models_X n \wedge h, E(\mathtt{acc}) \models_Y m \wedge X \cap Y = \emptyset) \longrightarrow \\
& \quad\quad (\exists Z. h', v \models_Z n+m \wedge r = r_{\mathtt{Rev4}}(n))\}
\end{array}\right]$$

Table 2
Specification table for class REV.

allocated space):

$$\rhd \mathsf{REV.}\mathtt{Rev}_2([a,b]) : \{\forall\ n\ X\ m\ Y.\ h, E(a) \models_X n \wedge h, E(b) \models_Y m \wedge X \cap Y = \emptyset \longrightarrow r_{\mathsf{Frames}} = n+1\}$$

In Rev_3, the method-level tail recursion is converted into a method-internal loop and the redundant field is eliminated, resulting in a program whose execution only requires a single frame.

$$\rhd \mathsf{REV.}\mathtt{Rev}_3([a,b]) : \{\forall\ n\ X\ m\ Y.\ h, E(a) \models_X n \wedge h, E(b) \models_Y m \wedge X \cap Y = \emptyset \longrightarrow r_{\mathsf{Frames}} = 1\}$$

The verification of the three specifications follows the same strategy as before. This is an overall optimisation for the resource algebra Frames, but we verified the intermediate steps using a more informative product resource algebra, Frames $\times$ MethCntsAll $\times$ Time $\times$ Heap, where MethCntsAll is similar to the MethCnts algebra shown in Table 1 except that only the *size* of the multisets is considered (thus we sum calls to all methods), and we stipulate that size(LIST) $= 1$. We obtained the following resource tuples which show the costs in terms of of the length n of the (first) input list.

	r_{Frames}	$r_{\mathsf{MethCntsAll}}$	r_{Times}	r_{Heap}
$r_{\texttt{App}}(n) \equiv$	$(n+1,$	$n+1,$	$22n+22$	$1)$
$r_{\texttt{Rev1}}(n) \equiv$	$(n+1,$	$n(n+1)/2,$	$11n^2+29n+11$	$n)$
$r_{\texttt{Rev2}}(n) \equiv$	$(n+1,$	$n+1,$	$20n+11$	$0)$
$r_{\texttt{Rev3}}(n) \equiv$	$(1,$	$1,$	$18n+11$	$0)$

Under the lexicographic order for the product algebra, both steps are optimising. To preserve datatype representation conditions across method calls, we need stronger invariants in the specifications than shown above. The full specification table is given in Table 2, which also contains an invariant of the loop `rev3`.

5 Static semantics

A static resource algebra $\mathcal{S}$ is defined as in Def. 2.1, except that the program constructors can depend on the typing context only; in particular, the method operator does not depend on the values of its arguments. For a fixed signature the judgment $\Gamma \vdash e : t, s$ assigns type t and effect s to Grail expression e in a straightforward way; an example is the rule for *if* expressions:

$$\frac{\Gamma \vdash e \ : \ \text{bool},\ s_e \qquad \Gamma \vdash e_1 \ : \ t,\ s_1 \qquad \Gamma \vdash e_2 \ : \ t,\ s_2}{\Gamma \vdash \mathsf{if}\ e\ \mathsf{then}\ e_1\ \mathsf{else}\ e_2 \ : \ t,\ s_e + \mathcal{S}^{\mathsf{if}} + s_1 + s_2}$$

Notice that this is slightly different from usual type and effect systems, where the effect on both branches would be the same. As well as rules for each expression, we have a sub-effecting rule which uses the ordering from $\mathcal{S}$. See Appendix A.4 for the full listing.

A number of the standard properties hold, culminating in subject reduction. All proofs are standard and hence omitted. First, it is immediate to show that our system is conservative w.r.t. the effect-free system defined, as usual, by erasure. A canonical forms property holds as follows, from which we observe that every value has *constant* effect.

Fact 5.1 (Canonical forms) *Assume that* $\Gamma \vdash v \ : \ t,\ s$*; then:*

- *if* $t = \text{int}$ *then* $v = i$ *and* $s = \mathcal{S}^{\mathsf{int}}$.
- *if* $t = \text{unit}$ *then* $v = ()$ *and* $s = 0$.
- *if* $t = C$ *then either* $v = \mathsf{null}_C$ *and* $s = \mathcal{S}^{\mathsf{null}}$ *or* $v = l_C$ *and* $s = 0$.

Weakening is admissible as well as a specialised form of substitution, in which arguments play the role of variables and the effect is increased accordingly. This generalises *value* substitution in type and effects systems, where substitution holds because values are *pure* (i.e. have a zero effect).

We now introduce a generalisation of the well-known relation between static effects and dynamic traces, which is key in the statement and proof of type soundness:

Definition 5.2 Given $\mathcal{S}$ and $\mathcal{R}$, an *approximation* is a relation $\sqsubseteq$ included in $S \times R$, which reads "static effect s approximates dynamic resource r", with the following properties:

(i) $0^{\mathcal{S}} \sqsubseteq 0^{\mathcal{R}}$ and for every constructor c, it holds $\mathcal{S}^c \sqsubseteq \mathcal{R}^c$.
(ii) If $s \sqsubseteq r$, then $\mathcal{R}^{\mathsf{meth}}_{C,m,\overline{a}}(s) \sqsubseteq \mathcal{R}^{\mathsf{meth}}_{C,m,\overline{a}}(r)$.
(iii) $s_1 \sqsubseteq r_1 \wedge s_2 \sqsubseteq r_2$ entails $s_1 +^{\mathcal{S}} s_2 \sqsubseteq r_1 +^{\mathcal{R}} r_2$;
(iv) $s \sqsubseteq r$ entails $s +^{\mathcal{S}} s' \sqsubseteq r$.

For example, w.r.t. the $\mathsf{MethGuard}$ algebra, its static approximation SG is $\langle \{\{\mathsf{tt}\}, \{\mathsf{tt}, \mathsf{ff}\}\}, \{\mathsf{tt}\}, \cup, \leq_{\mathsf{SG}} \rangle$, where the latter is defined as $b \leq_{\mathsf{SG}} b'$ iff $b = \{\mathsf{tt}\}$ or $b = b' = \{\mathsf{tt}, \mathsf{ff}\}$. The approximation relation is $\in^{-1}$, which trivially satisfies the conditions above.

Let $E : \Gamma$ be the usual correspondence between environment and typing. Further say that h is well-typed if $\Sigma(C.f) = t$ implies $h(l_C).f : t$. Finally, we say that a signature is *well-typed* if for all $g, m \in \Sigma$ it holds

$$\Sigma(g) = t_1 \times \cdots \times t_n \to t, s \Longrightarrow x_1 : t_1 \ldots x_n : t_n \vdash \mathit{body}_g : t, s$$
$$\Sigma(C.m) = t_1 \times \cdots \times t_n \to t, s \Longrightarrow x_1 : t_1 \ldots x_n : t_n \vdash \mathit{body}_{C,m} : t, s$$

Theorem 5.3 (Type soundness) *Assume a well-typed signature, $\mathcal{S}$ and $\mathcal{R}$ as above. Suppose further that $\Gamma \vdash e \ : \ t, \ s$ and for a well typed heap h it holds $E \vdash h, e \Downarrow h', v, r$ and $E : \Gamma$; then $\Gamma \vdash v \ : \ t, \ \mathit{scost}(v)$ and $s \sqsubseteq r$.*

The above result ensures the consistency of the operational semantics with the type system; it is a basis for approximating dynamic measurements using type checking instead of theorem proving.

6 Conclusions

We have presented a framework and methodology for *optimisation validation*, based on generic forms of dynamic and static resource costs. We have formalised most of the setting in Isabelle/HOL, particularly including the soundness and completeness of the program logic, which was applied to validate some specific optimisations in Sect. 4.

One can argue against our approach in various ways. For example, validating optimisation with disregard of behavioural equivalence seems pointless (often, the empty program is the ultimate optimisation). Yet, we see resource improvement validation as orthogonal to translation validation; in some settings one may check both things. Optimising compilers usually use heuristics to decide what to do with code, but in some cases a sequence of transformations may not actually result in improvement even if correctness is preserved; the optimisation is then pointless. In others settings, such as our PCC application, the safety policy that we care most about is captured by our resource consumption notion and so resource usage preservation is more crucial than functional equivalence.

Related Work. By now there is an extensive literature on verifying compiler correctness and optimisations (e.g. [10,19,9]), but as far as we know, no previous work on general methods for verifying that optimisations in fact improve resource usage. Specific instances of machine checked correctness proofs have been pursued, for example, w.r.t. dead code elimination [8], *tokenization* and *componentisation* transformations [12]. A related general approach is Sands' *Improvement Theory* [24], a specialisation of the standard theory and reasoning principles of observational equivalence in which basic observations include some intensional information about computational cost, extended to space improvements for effects-free call-by-need languages in [14].

Our direction was inspired by *translation validation* (TV) [22], implemented in the automatic TVOC tool [4] which passes verification conditions to a theorem prover. TV subsumes Rinard's *credible compilation* [23], which requires full code instrumentation. Necula [21] demonstrates TV based on symbolic execution in the context of the GNU C compiler intermediate language. As with Rinard, he uses simulation of execution paths, but instead of compiler annotation a constraint-based algorithm heuristically tries to infer a simulation.

Several other researchers have considered *program logics* which go beyond traditional functional specification. For example, in [26] a generic Hoare calculus for reasoning about computational monads is given. With a similar aim as us, Denney and Fischer [11] introduce a framework for safety policies: given a semantically defined safety property (such as "no division by zero") and an operational semantics, the aim is to derive specialised Hoare rules to enforce the property; however, for "stateful" properties, such as memory writes limits, the approach becomes technically quite involved.

Since we consider optimisation from one program to another, a natural approach might be to use a logic which relates two programs at once. In Benton's relational Hoare logic [5] judgements $\{R\}\ c_1 \sim c_2\ \{S\}$ refer to the execution of two (possibly) different programs c_1 and c_2 while the pre- and post-conditions are relations over states. A similar logic is used in Rinard's report [23]. In both cases, proofs of soundness are included while completeness is not examined.

Elsewhere, forms of *cost algebras* and *partial orders* similar to ours are considered for analysis of resource consumptions, e.g., [18,13] and optimisation [20]. General static analysis techniques which have similarities with the setup of our type and effect system include [15,27]. There is also work on specific static analyses for different notions of resource usage, beyond the scope of this brief overview.

Future work. There are several avenues for pursuing this work. First, there is further work possible on optimisation by considering finer-grained transformations individually, perhaps by generalising Improvement Theory to resource algebras. Second, it would be ideal if our static analysis was able to validate optimisations directly and avoid the need for the program logic: this is in fact possible in restricted (e.g. boolean) domains, but further assumptions are needed in the general case. To scale our techniques to routine application we would need either an automatic technique based on the type system or much better automatic assistance for using the program logic than the one provided by Isabelle/HOL. Endowing relational Hoare logics with a notion of resource algebra seems also a swift way to combine semantics preservation with optimisation validation.

Finally, the considerable generality of resource algebras allows examples that are less directly related to optimisation, but useful for validating other safety properties (including correspondence properties in protocols, or resource usage analysis in the sense of [17]), and we would like to apply our general techniques to those examples too.

The Isabelle sources for the core logic (and much more) are available at `http://www.tcs.ifi.lmu.de/~hwloidl/mrg/MRG-infra-0805.tgz` The examples presented in this paper can be downloaded from `http://homepages.inf.ed.ac.uk/amomigl1/papers/cocv06.tar`.

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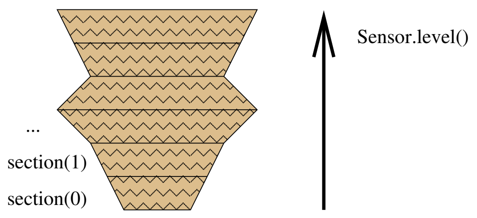
A Appendix

A.1 Optimisation of method call frequency

As well as standard optimisations, our framework can be used to validate optimisations which are custom specified for a particular application.

Consider the hypothetical scenario of a process-control application where there is an irregularly shaped chemical tank (illustrated opposite) whose contents must be carefully monitored to ensure sufficient reagent. When the amount reaches a critical low point, the reaction must be temporarily halted while the tank refills.



An embedded controller runs a program which monitors the level gauge. A suitable notion of optimisation in this setting would be to transform the program into one which checks the tank level more frequently, so reducing the latency between noticing a tank empty condition and triggering the refill cycle. Thus the frequency of invoking the `Sensor.level()` method is a suitable resource measure. Using the same methodology as previously, and with the resource algebra MethFreq, we can validate the transformation of a naive implementation of a program which calculates the amount of reagent into the tank into a better one which checks the level more frequently.

Below is the full listing of the example program (naive version) The method `calc(n)` calculates the amount of reagent left in `n` sections of the tank. It is invoked from the `runloop` method, which is supposed to be the process control loop; in reality, this loop would be run indefinitely and involve other tasks besides the level calculation.

```
class ChemCalc {
   field static int alarm
   field static int[] section
   field static int critical_amount
   method static void runloop() =
   let
     val n = 1000
```

```
    fun raise_alarm () = putstatic <int ChemCalc.alarm> 1
    fun loop_check(int n) = if n>0 then loop(n) else ()
    fun loop(int n) =
    let
      val chem_level = invokestatic <int Sensor.level()> ()
      val chem_amount = invokestatic <int ChemCalc.calc(int)> (chem_level)
      val n = sub n 1
      val critical = getstatic <int ChemCalc.critical_amount>
    in
      if chem_amount < critical then raise_alarm() else loop_check(n)
    end
  in loop_check(n) end
  method static int calc(int n) =
  let
    val a = 0
    fun sumup(int n, int a) =
    let
        val cs = getstatic <int[] ChemCalc.section>
        val x = get cs n
        val a = add x a
        val n = sub n 1
     in
        sumup_check (n,a)
     end
     fun sumup_check(int n, int a) =
         if n>0 then sumup(n,a)
                else a
  in sumup_check(n,a) end
```

Numerous optimisations are possible in this example to increase the rate of testing the sensor level. For example, we might sum up the section sizes only until we find out that the critical level has been safely exceeded. Or (supposing the dimensions of the tank are fixed during the run of the process), we may calculate the sums for the container sections in advance to avoid looping over the `section` array each time we test the sensor level. We have not yet undertaken the formal verification of this example, as it goes slightly beyond our formal presentation of the logic because it makes use of arrays; however, the extension is straightforward.

A.2 Operational semantics rules

The operational semantics rules below use the following notations for heaps:

$h(l)$	class name of object at l,
$h(l).f$	field lookup of value at f in object at l,
$h[l.f \mapsto v]$	field update of f with v at l.

Argument evaluation in an environment E is defined by $eval_E(x) = E(x)$ and $eval_E(v) = v$, while costs are defined by $cost() = cost(l) = 0$, $cost(\mathsf{null}) = \mathcal{R}^{\mathsf{null}}$, $cost(i) = \mathcal{R}^{\mathsf{int}}$, and $cost(x) = \mathcal{R}^{\mathsf{var}}$. The function $\mathsf{fields}(C)$ returns the sequence $\overline{f}$ of fields in the class C, while $inival_{f_i}$ denotes the initial value of the field f_i. For functions and methods, we write $body_g$ and $body_{C,m}$ to stand for the definition of g and $C.m$, respectively.

$$\frac{a \neq l_C}{E \vdash h, a \Downarrow h, eval_E(a), cost(a)}$$

$$\frac{E \vdash h, a \Downarrow h, v_a, r_a \qquad E \vdash h, a' \Downarrow h, v'_a, r'_a}{E \vdash h, \mathsf{prim}\ a\ a' \Downarrow h, \mathsf{prim}(v_a, v'_a), r_a + r'_a + \mathcal{R}^{\mathsf{prim}}}$$

$$\frac{l = \mathsf{freshloc}(h) \qquad \mathsf{fields}(C) = \overline{f}}{E \vdash h, \mathsf{new}\ C \Downarrow h[l.f_i \mapsto inival_{f_i}], l, \mathcal{R}^{\mathsf{new}}_C} \qquad \frac{E \vdash h, x \Downarrow l, h, r_x}{E \vdash h, x.f \Downarrow h, h(l).f, r_x + \mathcal{R}^{\mathsf{getf}}}$$

$$\frac{E \vdash h, x \Downarrow l, h, r_x \qquad E \vdash h, a \Downarrow v, h, r_a}{E \vdash h, x.f := a \Downarrow h[l.f \mapsto v], (), r_x + r_a + \mathcal{R}^{\mathsf{putf}}}$$

$$\frac{E \vdash h, e_1 \Downarrow h_1, (), r_1 \qquad E \vdash h_1, e_2 \Downarrow h_2, v, r_2}{E \vdash h, e_1\ ;\ e_2 \Downarrow h_2, v, r_1 + \mathcal{R}^{\mathsf{comp}} + r_2}$$

$$\frac{E \vdash h, e_1 \Downarrow h_1, v_1, r_1 \qquad E\langle x := v_1\rangle \vdash h_1, e_2 \Downarrow h_2, v, r_2}{E \vdash h, \mathsf{let}\ x = e_1\ \mathsf{in}\ e_2 \Downarrow h_2, v, r_1 + \mathcal{R}^{\mathsf{let}} + r_2}$$

$$\frac{E \vdash h, e \Downarrow h', \mathtt{1}, r_e \qquad E \vdash h', e_1 \Downarrow h'', v, r}{E \vdash h, \mathsf{if}\ e\ \mathsf{then}\ e_1\ \mathsf{else}\ e_2 \Downarrow h'', v, r_e + \mathcal{R}^{\mathsf{if}} + r}$$

$$\frac{E \vdash h, e \Downarrow h', \mathtt{0}, r_e \qquad E \vdash h', e_2 \Downarrow h'', v, r}{E \vdash h, \mathsf{if}\ e\ \mathsf{then}\ e_1\ \mathsf{else}\ e_2 \Downarrow h'', v, r_e + \mathcal{R}^{\mathsf{if}} + r}$$

$$\frac{E \vdash h, body_g \Downarrow h', v, r}{E \vdash h, \mathsf{call}\ g \Downarrow h', v, \mathcal{R}^{\mathsf{call}} + r}$$

$$\frac{eval_E(a_i) = v_i \qquad \{x_i := v_i\} \vdash h, body_{C,m} \Downarrow h', v, r}{E \vdash h, C.m(\overline{a}) \Downarrow h', v, (\Sigma_i\ cost(a_i)) + \mathcal{R}^{\mathsf{meth}}_{C,m,\overline{v}}(r)}$$

A.3 Program logic rules

$$\frac{e : P\ \in G}{G \rhd e : P} \qquad \frac{G \rhd e : P \quad P \longrightarrow Q}{G \rhd e : Q}$$

$$\frac{}{G \rhd a : \{h' = h\ \wedge\ v = eval_E(a)\ \wedge\ r = cost(a)\}}$$

$$\frac{}{\begin{array}{l} G \rhd \mathsf{prim}\ a_1\ a_2 : \{h' = h\ \wedge\ v = \mathsf{prim}(eval_E(a_1), eval_E(a_2))\ \wedge \\ \qquad\qquad\qquad\qquad r = cost(a_1) + cost(a_2) + \mathcal{R}^{\mathsf{prim}}\}\end{array}}$$

$$\frac{}{G \rhd \mathsf{new}\ C : \{v = \mathsf{freshloc}(h)\ \wedge\ h' = h[v.f_i \mapsto \mathit{initval}_{f_i}]\ \wedge\ r = \mathcal{R}^{\mathsf{new}}_C\}}$$

$$\frac{}{G \rhd x.f : \{h = h' \wedge (\exists l.E(x) = l \wedge v = h(l).f) \wedge r = \mathit{cost(x)} + \mathcal{R}^{\mathsf{getf}}\}}$$

$$\frac{}{G \rhd x.f := a\ :\ \{(\exists l.E(x) = l \wedge h' = h[l.f \mapsto \mathit{eval}_E(a)]) \wedge v = () \wedge r = \mathit{cost(x)} + \mathit{cost}(a) + \mathcal{R}^{\mathsf{putf}}\}}$$

$$\frac{G \rhd e_1 : P_1 \qquad G \rhd e_2 : P_2}{\begin{array}{c}G \rhd e_1\ ;\ e_2 : \{\exists\ h_1\, r_1\, r_2.\ P_1[E, h, h_1, (), r_1] \wedge P_2[E, h_1, h', v, r_2] \wedge \\ r = r_1 + \mathcal{R}^{\mathsf{comp}} + r_2\}\end{array}}$$

$$\frac{G \rhd e_1 : P_1 \qquad G \rhd e_2 : P_2}{\begin{array}{c}G \rhd \mathsf{let}\ x = e_1\ \mathsf{in}\ e_2 : \{\exists\ h_1\, v_1, r_1\, r_2.\ P_1[E, h, h_1, v_1, r_1] \wedge\ P_2[E[x := v_1], h_1, h', v, r_2] \wedge \\ r = r_1 + \mathcal{R}^{\mathsf{let}} + r_2\}\end{array}}$$

$$\frac{G \rhd e_1 : P_1 \qquad G \rhd e_2 : P_2 \qquad G \rhd e_3 : P_3}{\begin{array}{l}G \rhd \mathsf{if}\ e_1\ \mathsf{then}\ e_2\ \mathsf{else}\ e_3 : \{\exists\ h_1\, v_1\, r_1\, r_2.\ P_1[E, h, h_1, v_1, r_1] \wedge \\ \quad (v_1 = \mathsf{1} \Longrightarrow P_2[E, h_1, h', v, r_2]) \wedge \\ \quad (v_1 = \mathsf{0} \Longrightarrow P_3[E, h_1, h', v, r_2])\ \wedge\ r = r_1 + \mathcal{R}^{\mathsf{if}} + r_2\}\end{array}}$$

$$\frac{G \cup \{(\mathsf{call}\ g, P)\} \rhd \mathit{body}_g : P[E, h, h', v, \mathcal{R}^{\mathsf{call}}_g + r]}{G \rhd \mathsf{call}\ g : P[E, h, h', v, r]}$$

$$\frac{G \cup \{(C.m(\overline{a}), P)\} \rhd \mathit{body}_{C,m} : P[\{x_i := \mathit{eval}_E(a_i)\}, h, h', v, \mathcal{R}^{\mathsf{meth}}_{C,m,\mathit{eval}_E(\overline{a})}(r)]}{G \rhd C.m(\overline{a}) : P[E, h, h', v, r]}$$

A.4 Typing rules

$$\frac{}{\Gamma \vdash a\ :\ \mathit{type}_\Gamma(a),\ \mathit{scost}(a)} \qquad \frac{\Gamma \vdash e\ :\ t,\ s \qquad s \leq s'}{\Gamma \vdash e\ :\ t,\ s'}$$

$$\frac{\Gamma \vdash a_1\ :\ t_1,\ s_1 \qquad \Gamma \vdash a_2\ :\ t_2,\ s_2 \qquad \Sigma(\mathsf{prim}) = t_1 \times t_2 \to t_3}{\Gamma \vdash \mathsf{prim}\ a_1\ a_2\ :\ t_3,\ s_1 + s_2 + \mathcal{S}^{\mathsf{prim}}}$$

$$\frac{}{\Gamma \vdash \mathsf{new}\ C\ :\ C,\ \mathcal{S}^{\mathsf{new}}_C} \qquad \frac{\Gamma \vdash x\ :\ C,\ s \qquad \Sigma(C.f) = t}{\Gamma \vdash x.f\ :\ t,\ s + \mathcal{S}^{\mathsf{getf}}}$$

$$\frac{\Gamma \vdash x\ :\ C,\ s_x \qquad \Gamma \vdash a\ :\ t,\ s_a \qquad \Sigma(C.f) = t}{\Gamma \vdash x.f := a\ :\ \mathrm{unit},\ s_x + s_a + \mathcal{S}^{\mathsf{putf}}}$$

$$\frac{\Gamma \vdash e_1\ :\ \mathrm{unit},\ s_1 \qquad \Gamma \vdash e_2\ :\ t_2,\ s_2}{\Gamma \vdash e_1\ ;\ e_2\ :\ t_2,\ s_1 + \mathcal{S}^{\mathsf{comp}} + s_2} \qquad \frac{\Gamma \vdash e_1\ :\ t_1,\ s_1 \qquad \Gamma, x : t_1 \vdash e_2\ :\ t_2,\ s_2}{\Gamma \vdash \mathsf{let}\ x = e_1\ \mathsf{in}\ e_2\ :\ t_2,\ s_1 + \mathcal{S}^{\mathsf{let}} + s_2}$$

$$\frac{\Gamma \vdash e \ : \ \text{bool},\ s_e \qquad \Gamma \vdash e_1 \ : \ t,\ s_1 \qquad \Gamma \vdash e_2 \ : \ t,\ s_2}{\Gamma \vdash \mathsf{if}\ e\ \mathsf{then}\ e_1\ \mathsf{else}\ e_2 \ : \ t,\ s_e + \mathcal{S}^{\mathsf{if}} + s_1 + s_2}$$

$$\frac{\Sigma(g) = t_1 \times \cdots \times t_n \to t, s}{\Gamma \vdash \mathsf{call}\ g : t, \mathcal{S}^{\mathsf{call}} + s}$$

$$\frac{\Gamma \vdash a_i \ : \ t_i,\ s_i \qquad \Sigma(m) = t_1 \times \cdots \times t_n \to t, s}{\Gamma \vdash C.m(\overline{a}) \ : \ t,\ \Sigma_i s_i + \mathcal{S}^{\mathsf{meth}}_{C,m,\overline{a}}(s)}$$

Argument typing is defined by $\mathit{type}_\Gamma(x) = \Gamma(x)$, $\mathit{type}_\Gamma(i) = \text{int}$, $\mathit{type}_\Gamma() = \text{unit}$, $\mathit{type}_\Gamma(l_C) = C$ and $\mathit{type}_\Gamma(\mathsf{null}_C) = C$; argument static costs are defined by $\mathit{scost}(\mathsf{null}_C) = \mathcal{S}^{\mathsf{null}}$, $\mathit{scost}(i) = \mathcal{S}^{\mathsf{int}}$, $\mathit{scost}(x) = \mathcal{S}^{\mathsf{var}}$ and $\mathit{scost}(()) = \mathit{scost}(l) = 0$.